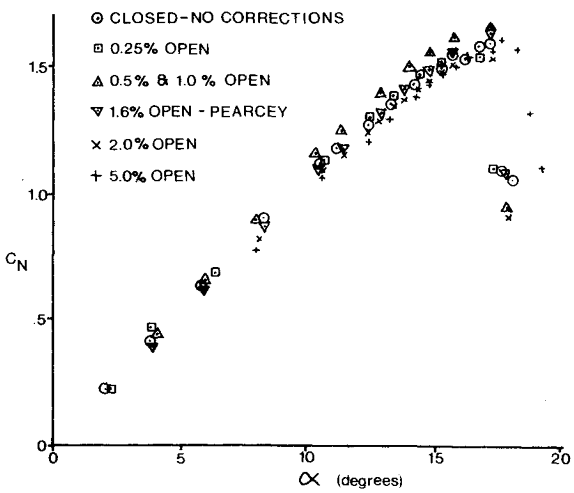

Fig. 3 C_N **vs** α**—various slot sizes.**

stall increase slightly until 0.5%–1% open after which they decrease, falling below the uncorrected closed values for 2% open (the 1.6% open configuration was of the type suggested by Pearcey[6] where the upper and lower slots are half the size of the middle ones). Increasing from 2% to 5% open continues to decrease C_N but also causes a sharp increase in the stall angle.

From these results it appears that the results for the 2% open walls are closest to the corrected values and a comparison is shown in Fig. 4. Whilst C_N is still slightly high (about 2%) for the slotted configuration at high angles of attack, the stall angle is correct. With larger slots the C_N values could be lowered (decreasing blockage) but would result in a greater stall angle (increasing stream curvature).

Quarter chord pitching moments for both the closed with corrections and the 2% open configuration remained zero up to stall at which point they both went negative.

The above results agree closely with those of Pearcey,[6] who found for his tunnel configuration that four slots 1.6% open gave zero blockage and lift interference.

Both Pearcey's and the current slot openings are considerably above values predicted by theory[5] which indicates that for an ideal (i.e., zero viscous effects at the slots) tunnel of the type used, blockage should have been eliminated and lift interference considerably reduced with the walls 0.002% open. The theory does not predict the initial move away from "correct" data for small slot openings that the current results reveal.

These vast discrepancies between experimental results and theoretical predictions render the latter suspect. Even consideration of the viscous effects for the flow through the slots is unlikely to provide good agreement. A more complex representation of the boundary conditions at the slotted walls

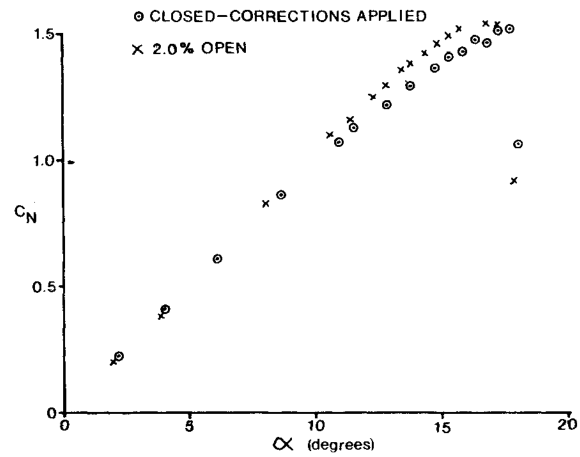

Fig. 4 C_N **vs** α**—closed and 2.0% open.**

is probably required, the linearized condition being insufficient particularly for large models at high angles of attack.

Conclusions

For the particular configuration tested, i.e., a 4-ft chord airfoil in the Texas A&M 7 ft × 10 ft low speed tunnel, it was found that corrections due to the tunnel boundaries could not be completely eliminated by using slotted sidewalls in the test section. However, by using four slots, 2% open, lift interference could be removed (i.e., stall occurred at the correct angle) and blockage reduced such that at stall the normal force coefficient was only 2% high. This is considered to be a distinct improvement over uncorrected results from a closed tunnel where α stall is 2.5% low and C_N max is 7% high. Based on these results, a series of oscillatory tests using the 4-ft chord airfoil will be undertaken with the test section sidewalls 2% open.

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Separating Laminar Boundary Layers with Prescribed Wall Shear

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Introduction

KELLER and Cebeci[1] have presented some solutions computed by a finite-difference method to what are termed "inverse problems" in the study of the boundary-layer equations for laminar, two-dimensional, incompressible flow. An inverse problem is one in which the streamwise distribution of wall shear is given, while the pressure distribution is an unknown function to be determined. This is in contrast to a "standard problem," in which the pressure distribution is given while the wall shear distribution is unknown.

Inverse problems are of particular interest in the context of separation, since solutions of standard problems exhibit singular behavior in the vicinity of separation which prevents continuation

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beyond that point of the downstream integration of the parabolic equations.[2] Hopefully, an inverse problem in which the prescribed wall shear distribution is nonsingular should enable a smooth passage through separation to be made. However, the numerical method used in Ref. 1 failed shortly ahead of separation in the two cases considered.

The present Note briefly describes an alternative numerical procedure for solving the inverse problem, and compares results obtained using this procedure with some results from Ref. 1. In addition, results are presented for a case in which the present procedure has enabled integration to be continued smoothly through both separation and reattachment.

Equations

The boundary-layer equation in the form due to Görtler[3] is used, namely

$$\frac{\partial^3 f}{\partial \eta^3}+f\frac{\partial^2 f}{\partial \eta^2}+\beta\left[1-\left(\frac{\partial f}{\partial \eta}\right)^2\right]=2\xi\left[\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \eta\,\partial \xi}-\frac{\partial^2 f}{\partial \eta^2}\frac{\partial f}{\partial \xi}\right] \quad (1)$$

where $f(\xi,\eta)$ is the dimensionless stream function, ξ is the scaled streamwise variable, and η is the similarity variable. Also $\beta(\xi)$ is the pressure gradient parameter, related to the local velocity outside the boundary layer, $u_1(\xi)$, by

$$\beta = 2d(\ln u_1)/d(\ln \xi) \quad (2)$$

In the inverse problem the relevant boundary conditions are

$$\text{Inner: } f(\xi,0)=\partial f/\partial\eta(\xi,0)=0,\ \partial^2 f/\partial\eta^2(\xi,0)=S(\xi) \quad (3a)$$

$$\text{Outer: } \partial f/\partial\eta(\xi,\eta)\to 1 \quad \text{as} \quad \eta\to\infty \quad (3b)$$

Here $S(\xi)$ is the prescribed distribution of nondimensional wall shear. These four boundary conditions correctly determine the third-order equation when $\beta(\xi)$ is treated as unknown.

Solution

The method of solution adopted is closely related to the differential-difference method used by Smith and Clutter[4] to solve the standard problem, but with an improved shooting technique to solve the resulting set of ordinary differential equations. Thus, the range of integration with respect to ξ is divided into a suitable number of intervals, and derivatives with respect to ξ in Eq. (1) at each station ξ_n are replaced by 3- or 4-point Lagrange backward-difference formulas. This results in an ordinary differential equation for $f_n(\eta)$ at each ξ_n. These equations are solved successively downstream, starting at $\xi = 0$. The integrations with respect to η use Eqs. (3a) as initial conditions, together with an estimated value of β, as the first cycle of an iterative procedure which progressively improves the value of β until the outer boundary condition, Eq. (3b), is satisfied at a sufficiently large value of η, say η_∞.

The procedure used to improve the estimated value of β is as follows. In the ordinary differential equation to be solved, f is treated as a function of both η and β. Then partial differentiation of the equation with respect to β yields an additional third-order equation for the variation $F(\eta)\equiv \partial f/\partial\beta(\eta)$. This equation is integrated simultaneously with the original equation, with initial conditions

$$F(0)=dF/d\eta(0)=d^2F/d\eta^2(0)=0 \quad (4)$$

Applying Newton's method, the first-order correction to β required to satisfy Eq. (3b) is

$$\delta\beta=[1-df/d\eta(\eta_\infty,\beta)]/dF/d\eta(\eta_\infty,\beta) \quad (5)$$

In general the procedure converges rapidly, but for small values of the step length $\Delta\xi$ the solution becomes exceedingly sensitive to the value of β, and double-precision arithmetic is required. A similar difficulty occurs in the solution of the standard problem.[4] Fourth-order Runge-Kutta integration is used to solve the ordinary differential equations.

Results

Case I $S(\xi) = 0.46960(1-\xi),\ \xi \geqq 0$

This is Case A of Ref. 1, and corresponds to a boundary layer originating with zero pressure gradient at $\xi = 0$, and separating at $\xi = 1$. A comparison of the distribution of $\beta(\xi)$ computed by the present method with corresponding results from Keller and Cebeci[1] is given in Table 1. For the present results the 3-point streamwise difference scheme was used with $\Delta\xi = 0.05$, and the crosswise integration step $\Delta\eta$ was 0.2. Errors in the crosswise integrations are considered negligible. The results of Ref. 1 were computed using the same value of $\Delta\xi$ by means of a finite-difference scheme having the same order of error in ξ-discretization [i.e., $0(\Delta\xi^2)$] as the present scheme, and h^2-extrapolation was applied to calculations using $\Delta\eta = 0.5$ and 0.25 to reduce the η-discretization error to $0(\Delta\eta^3)$. The present results and those of Ref. 1 differ by 0.1% at most for $\xi \leqq 0.9$, beyond which point the method of Ref. 1 failed.

Table 1 Distribution of $\beta(\xi)$ for Case I

ξ	Present results β	Keller and Cebeci[1] β	Series, Eq. (6) β
0.00	0.00000	−0.00003	0.00000
0.20	−0.08432	−0.08424[a]	−0.08429
0.40	−0.15436	−0.15418[a]	−0.15428
0.60	−0.20776	−0.20747	−0.20798
0.80	−0.23949	−0.23940	−0.24229
0.90	−0.24349	−0.24376	−0.25085
0.95	−0.24088	—[b]	−0.25269
1.00	−0.23353	—	−0.25278

[a] Numerical errors in Ref. 1 have been corrected.
[b] Iteration procedure diverged.

As an additional check, Table 1 also includes values of β calculated using the fifth-degree power series of Görtler,[3] which may be straightforwardly inverted to obtain β as a power series in ξ for a given polynomial distribution of $S(\xi)$. For the present case the series becomes

$$\beta = -0.454880\xi+0.162855\xi^2+0.018834\xi^3+0.012834\xi^4+0.007582\xi^5+\cdots \quad (6)$$

For $\xi < 0.6$ the series solution agrees well with both the present results and those of Ref. 1, but thereafter there is a progressively increasing difference. A major part of this discrepancy is probably due to retention of insufficient terms in the series, which converges only slowly, if at all, at $\xi = 1$.

Case II $S(\xi) = 0.46960(1-\xi)(1-0.52649\xi),\ \xi \geqq 0$

This prescribed distribution of wall shear is shown in Fig. 1. Separation is again at $\xi = 1$, while there is a minimum value of S of -0.05 at $\xi \simeq 1.45$, followed by re-attachment at $\xi \simeq 1.9$. Computed distributions of $\beta(\xi)$ are shown in Fig. 2, using both

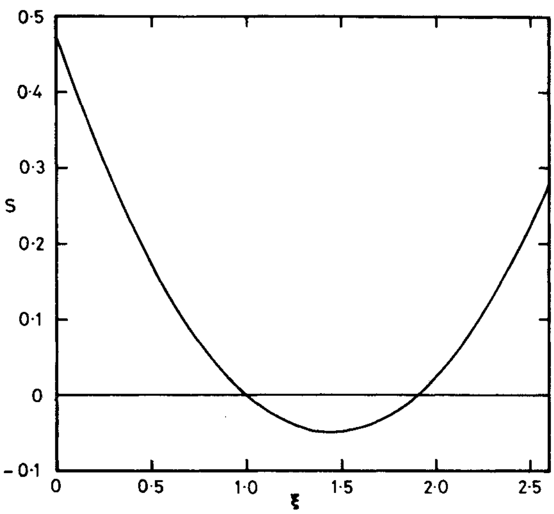

Fig. 1 Distribution of wall shear $S(\xi)$ prescribed in Case II.

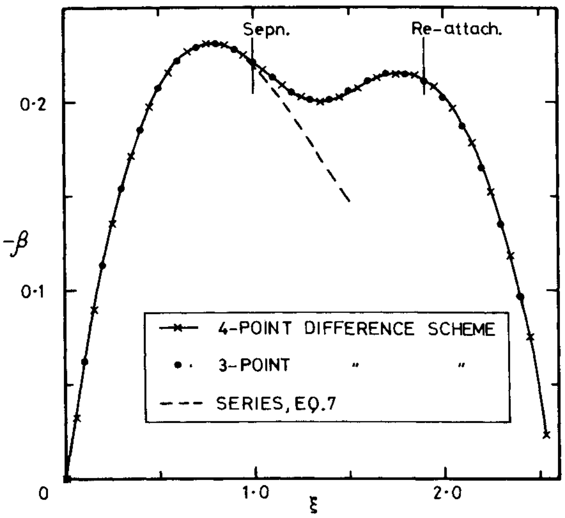

Fig. 2 Distribution of pressure gradient parameter $\beta(\xi)$ computed in Case II.

3- and 4-point ξ-difference schemes with $\Delta\xi = 0.1$, $\Delta\eta = 0.2$. Despite the existence of reversed flow for $1 < \xi < 1.9$, no computational difficulties were experienced and a smooth passage through both points of zero wall shear was achieved. The small difference between the 3-point and 4-point scheme results, which have discretization errors of order $(\Delta\xi)^2$ and $(\Delta\xi)^3$, respectively, is some indication that such errors are not large.

Also shown in Fig. 2 is the corresponding distribution of $\beta(\xi)$ calculated from the inverted Görtler series, which in this case is

$$\beta = -0.694370\xi + 0.651737\xi^2 - 0.204289\xi^3 + 0.037323\xi^4 - 0.009440\xi^5 + \cdots \qquad (7)$$

Equation (7) is in good agreement with the numerical results for $\xi < 1$, but thereafter rapidly diverges from them. The radius of convergence of the series is not known (although the behavior of the coefficients suggests that it may be greater than unity), and no conclusions can be made from this comparison concerning the accuracy of the numerical results.

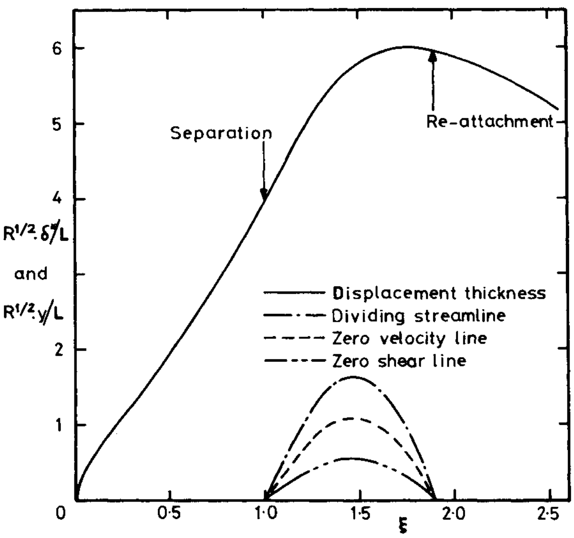

Fig. 3 Distribution of displacement thickness δ^* and the dividing streamline, zero velocity line and zero shear line for Case II.

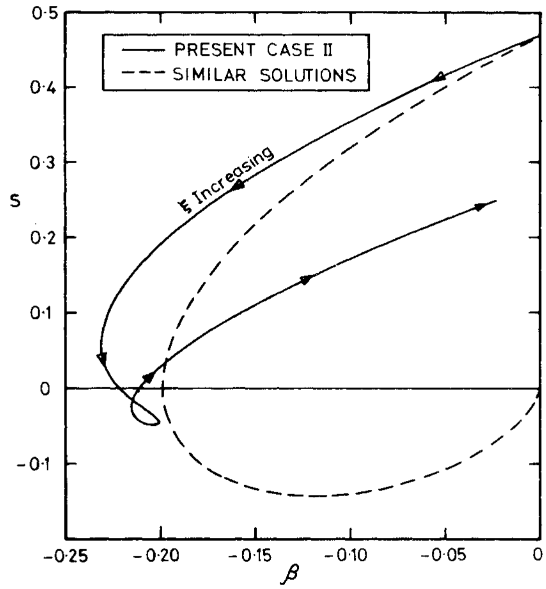

Fig. 4 $S-\beta$ locus for Case II compared with that for similar solutions.

The distribution of displacement thickness δ^* and the locations of the dividing streamline, zero velocity line and zero shear line are shown in Fig. 3. Here, R is a reference Reynolds number, L is a reference length, and y is distance normal to the surface.

The extent of departure from local similarity is shown in Fig. 4, which compares the computed $S-\beta$ locus with that corresponding to similar solutions [that is, solutions of Eq. (1) with β constant and the right-side identically zero].

Conclusions

There is no evidence of singular behavior at separation in either of the two cases considered, and indeed a criterion for regular separation quoted by Brown and Stewartson,[2] namely the vanishing at separation of the fourth derivative with respect to y of the tangential velocity component evaluated at $y = 0$, is satisfied to within the computational accuracy in both cases. It therefore appears that a regular prescribed wall shear distribution may in general lead to entirely regular behavior at separation.

Additionally, there is no evidence of numerical instability in the reversed flow region in Case II, although it has been suggested[2] that instability is to be expected. Further computations are required to establish whether cases with more extensive regions of reversed flow will exhibit unstable behavior analogous with that experienced by Catherall and Mangler[5] in an alternative type of inverse problem.

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